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Maximum genus and maximum nonseparating independent set of a 3-regular graph

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Abstract

A set $J \subseteq V$ is called a *nonseparating independent set* (*nsis*) of a connected graph $G = (V, E)$, if J is an independent set of G, i.e., $E \cap \{uv \mid \forall u, v \in J\} = \emptyset$, and $G - J$ is connected. We call $z(G) = \max_J\{|J| \mid J$ is an nsis of $G\}$ the *nsis number* of G. Let G be a 3-regular connected graph; we prove that the maximum genus, denoted by $\gamma_M(G)$, of G is equal to $z(G)$. Then, according to this result, some new characterizations of the maximum genus $\gamma_M(G)$ are obtained.

1. Introduction

Graphs considered here are finite and undirected, but multiple edges and loops are permitted. Terminology and notation not defined in this paper will generally conform to that in [1].

Let $G = (V, E)$ be a graph where V, E are the set of vertices and edges of G, respectively. For a vertex $u \in V(G)$, usually denote by $N_G(u)$ the neighborhood of u in G. For $A \subseteq V$, $G - A$ (or $G \backslash A$) is the graph obtained from G by deleting the vertices in A and deleting the edges incident to the vertices in A. A set $J \subseteq V$ is called a *nonseparating independent set* (*nsis*) of G, if J is an independent set of G, i.e., $E \cap \{uv \mid \forall u, v \in J\} = \emptyset$, and $G - J$ is connected. We will still use the notation appearing in other papers (see [8]), let $z(G) = \max_J\{|J| \mid J$ is an nsis of $G\}$ be called the nsis number of G. We say J to be a *maximum nsis* of G if J is an nsis of G, and $|J| = z(G)$. The problems of the (maximum) nsis in a graph have been well studied, especially for a 3-regular graph [7–10, 12].

The *maximum genus*, denoted by $\gamma_M(G)$, of a connected graph G is the largest genus of an orientable surface on which G admits a 2-cell embedding. We know [6]

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that $\gamma_M(G) \leqslant \lfloor \beta(G)/2 \rfloor$, where $\beta(G) = |E(G)| - |V(G)| + 1$ is the *Betti number* of G. If $\gamma_M(G) = \lfloor \beta(G)/2 \rfloor$, we say that G is *up-embeddable.* For a spanning tree T of G, a connected component L of $G - E(T)$ is called an *odd size component* if L has an odd number of edges; otherwise, L is called an *even size component.* Let $\xi(G,T)$ denote the number of odd size components of $G - E(T)$. We call $\xi(G) = \min_T\{\xi(G,T) \mid T$ is a spanning tree of $G\}$ the *Betti deficiency* of G. It is known ([11] or [4, Ch.12]) that the maximum genus $\gamma_M(G)$ and the up-embeddablity of a connected graph are characterized by the following theorem.

Theorem. *Let G be a connected graph. Then,*
(1) *G is up-embeddable if and only if $\xi(G) \leqslant 1$,*
(2) $\gamma_M(G) = [\beta(G) - \xi(G)]/2$.

We notice that $\gamma_M(G)$ is a topological invariant; in other words, if $H \simeq G$ ('$\simeq$' denotes the homeomorphic relation), then $\gamma_M(H) = \gamma_M(G)$.

Let G be a 3-regular connected graph. In Section 3, we prove that $\gamma_M(G)$ is equal to $z(G)$. Then by this fact, some new characterizations on $\gamma_M(G)$ are obtained, which include the generalization of a result in [8]. In Section 4, based on some known results of $z(G)$, some new lower bounds on $\gamma_M(G)$ are derived. In [10], the authors show that the nsis problem can be solved in polynomial time for graphs with no vertex degree exceeding three by reducing the problem to the matroid parity problem (for detail, see [5, 10]). Thus, we also obtain a polynomial-time algorithm to compute the maximum genus $\gamma_M(G)$ of a 3-regular graph. In [2] a polynomal-time algorithm for finding a maximum genus embedding of a graph is presented. But here, we do not investigate the algorithm in detail.

2. Lemmas

Before establishing our results, we need some lemmas.

Lemma 1. *Let G be a 3-regular connected graph. If there exists a vertex $u \in V(G)$ such that $G - u$ is connected and u is not incident to a loop, then $\xi(G - u) \geqslant \xi(G)$.*

Proof. Otherwise, assume that $\xi(G-u) < \xi(G)$. Since $G - u$ is connected, then there exists a spanning tree T' of $G - u$ such that $\xi(G - u, T') = \xi(G - u)$. Let $x \in N_G(u)$. Choose $T = T' \cup \{xu\}$ as a spanning tree of G. Because u is not incident to a loop, we have $\xi(G,T) \leqslant \xi(G - u, T') = \xi(G - u)$. Hence, it follows that $\xi(G) \leqslant \xi(G,T) \leqslant \xi(G - u, T') = \xi(G - u) < \xi(G)$. A contradiction ! □

Lemma 2. *Let G be a 3-regular connected graph and T be a spanning tree of G such that $\xi(G,T) = \xi(G)$. Let L be a connected component of $G - E(T)$. If $|E(L)| \geqslant 2$, then there exists $u \in V(G)$, u is not incident to a loop, such that $G - u$ is connected and $\xi(G - u) = \xi(G)$.*

Proof. Since G is 3-regular, then each connected component of $G-E(T)$ is a path or a circuit.

Case 1: *When L is a path.* Because $|E(L)|\geqslant 2$, let $L = v_1v_2\cdots v_s$, $s\geqslant 3$. Clearly, $G-v_2$ is connected and v_2 is not incident to a loop. By Lemma 1, $\xi(G-v_2)\geqslant\xi(G)$. On the other hand, choose $T' = T-v_2$ as a spanning tree of $G-v_2$. No matter whether L is an even size component or odd size component of $G-E(T)$, we may easily know that $\xi(G-v_2,T') = \xi(G,T)$. Then we have $\xi(G-v_2)\leqslant\xi(G-v_2,T') = \xi(G)$. Hence, it implies that $\xi(G-v_2) = \xi(G)$.

Case 2: *When L is a circuit.* Similarly, let $L = v_1v_2\cdots v_sv_1$, $s\geqslant 2$. By the same way as above, we see that $G-v_i$ is connected, v_i is not incident to a loop, and $\xi(G-v_i) = \xi(G)$ for any v_i, $1\leqslant i\leqslant s$.

Therefore, the proof of the lemma is complete. □

Lemma 3. *Let G be a 3-regular connected graph. If $\gamma_M(G)\neq 0$, then there exists a $u\in V(G)$, u is not incident to a loop, such that $G-u$ is connected and $\gamma_M(G-u) = \gamma_M(G)-1$.*

Proof. Let T be a spanning tree of G such that $\xi(G,T) = \xi(G)$. We deal with the following two cases.

Case 1: If there exists a connected component L of $G-E(T)$ such that $|E(L)|\geqslant 2$, then by Lemma 2, there exists $u\in V(G)$, u is not incident to a loop, such that $G-u$ is connected and $\xi(G-u) = \xi(G)$. Because G is 3-regular and u is not incident to a loop, we have $\beta(G-u) = \beta(G)-2$. Thus, we get that

$$\begin{aligned}\gamma_M(G-u) &= \frac{\beta(G-u)-\xi(G-u)}{2}\\ &= \frac{\beta(G)-2-\xi(G)}{2}\\ &= \gamma_M(G)-1.\end{aligned}$$

Hence, the conclusion of the lemma holds.

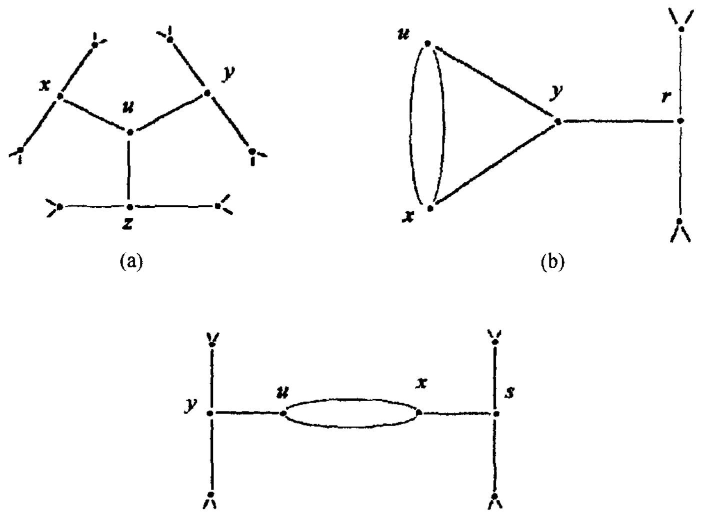
Case 2: If for any connected component L of $G-E(T)$, $|E(L)|\leqslant 1$, then we see that $\xi(G,T) = |E(G)|-|E(T)| = \beta(G)$, namely $\xi(G) = \xi(G,T) = \beta(G)$. Thus, $\gamma_M(G) = [\beta(G)-\xi(G)]/2 = 0$, which contradicts the assumption that $\gamma_M(G)\neq 0$.

From Cases 1 and 2, the lemma is proved. □

Notice that if $v\in V(G)$ is a vertex of degree 1 in G, then $\gamma_M(G-v) = \gamma_M(G)$. This fact will be used in the proof of the following lemma.

Lemma 4. *Let G be a 3-regular connected graph and let $\gamma_M(G)\neq 0$. Then, one of the following three cases is true.*

(A) *There exists a $u\in V(G)$ such that $G-u$ is connected without a vertex of degree* 1 *and $\gamma_M(G-u) = \gamma_M(G)-1$ (see Fig.* 1(a)).



(a) (b) (c)

Fig. 1.

(B) *There exist $u,x,y \in V(G)$, where ux is a multiple edge of G, and $yu, yx \in E(G)$, such that $G\backslash\{u,x,y\}$ is connected without a vertex degree of 1 and $\gamma_M(G\backslash \{u, x, y\}) = \gamma_M(G) - 1$ (see Fig. 1(b)).*

(C) *There exist $u,x \in V(G)$, where ux is a multiple edge of G, such that $G\backslash\{u,x\}$ is connected without a vertex of degree of 1 and $\gamma_M(G\backslash\{u, x\}) = \gamma_M(G) - 1$ (see Fig. 1(c)).*

Proof. Because $\gamma_M(G) \neq 0$, by Lemma 3 there exists a vertex $u \in V(G)$, u is not incident to a loop, such that $G - u$ is connected and $\gamma_M(G - u) = \gamma_M(G) - 1$. Let $N_G(u) = \{x, y, z\}$. Since u is not incident to a loop, we have $u \notin \{x, y, z\}$.

Case 1: *When* $|N_G(u)| = 3$ *or* 1. In this case, we have that either $x \neq y \neq z$ or $x = y = z$. It implies that $G - u$ has no vertex of degree of 1. Hence, the statement (A) holds.

Case 2: *When* $|N_G(u)| = 2$. Without loss of generality, let $x = z \neq y$, i.e., ux is a multiple edge of G. Let $s \in N_G(x)$, but $s \neq u$.

If $s = y$, because G is 3-regular and $G - u$ is connected, we get that $G\backslash\{u, x, y\}$ is connected and has no vertex of degree of 1. Because x is a vertex of degree of 1 in $G - u$, we know that $\gamma_M(G\backslash\{u,x,\}) = \gamma_M(G - u) = \gamma_M(G) - 1$. Similarly, because y is a vertex of degree 1 in $G\backslash\{u,x\}$, we have that $\gamma_M(G\backslash\{u,x,y\}) = \gamma_M(G\backslash\{u,x\}) = \gamma_M(G) - 1$. Thus, the statement (B) holds.

If $s \neq y$, by the same reasoning on the above cases, we easily get that the statement (C) holds. $\square$

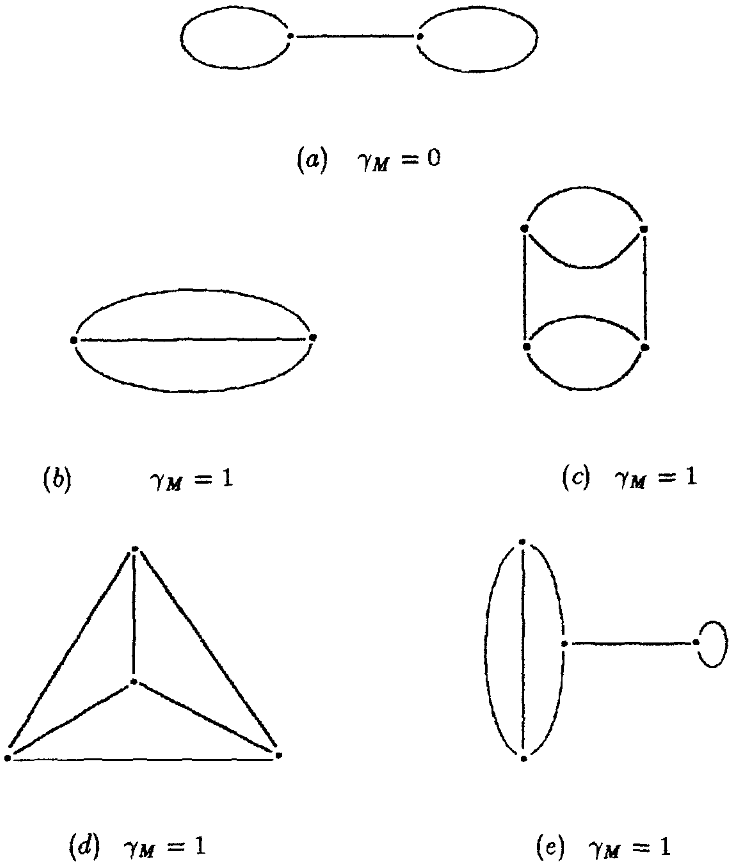


Fig. 2.

Lemma 5. *Let G be a 3-regular connected graph. Then there exists an independent set $J \subseteq V(G)$ of G such that $|J| = \gamma_{\mathrm{M}}(G)$ and $G - J$ is connected.*

Proof. Since G is 3-regular, $|V(G)|$ is even. We prove the lemma by induction on $|V(G)|$.

If $|V(G)| \leqslant 4$, we notice that G must be one of the graphs shown in Fig. 2(a)–(e). It is easy to check that the conclusion of the lemma holds for $|V(G)| \leqslant 4$.

Now we assume $|V(G)| \geqslant 6$, and assume that the theorem is true for all graphs with fewer than $|V(G)|$ vertices. When $\gamma_{\mathrm{M}}(G) = 0$, obviously the conclusion of the lemma is true. Now we assume $\gamma_{\mathrm{M}}(G) \neq 0$. Because $\gamma_{\mathrm{M}}(G) \neq 0$, by Lemma 4 one of the three statements (A), (B) and (C) of Lemma 4 holds. Now, we consider the three cases respectively.

Case 1: *When (A) of Lemma 4 holds.* Let G be as shown in Fig. 1(a). Let $N_G(x) = \{u, s_1, t_1\}$, $N_G(y) = \{u, s_2, t_2\}$ and $N_G(z) = \{u, s_3, t_3\}$ (it is allowed to have that some are the same among $s_1, t_1, s_2, t_2, s_3, t_3$). Let H be a topological subgraph of G and $H \simeq G - u$. Because $G - u$ is connected and has no vertex of degree 1, from $|V(G)| \geqslant 6$ we know that H is a 3-regular connected graph and that $\gamma_M(H) = \gamma_M(G-u) = \gamma_M(G) - 1$. Clearly, $|V(H)| < |V(G)|$. Thus, by the inductive hypothesis, there exists an independent set $J' \subseteq V(H)$ such that $H - J'$ is connected and $|J'| = \gamma_M(H)$. Choose $J = J' \cup \{u\} \subseteq V(G)$. Clearly, J is an independent set of G and $|J| = |J'| + 1 = \gamma_M(H) + 1 = \gamma_M(G)$. Since J' is an independent set of H, at least one of s_i and t_i ($i = 1,2,3$) does not belong to J'. Because $H - J'$ is connected, it follows that $G - J$ is connected. The conclusion of the lemma holds for G.

Case 2: *When* (B) of *Lemma* 4 *happens.* Let G be as shown in Fig. 1(b). Let $N_G(y) = \{u,x,r\}$ and $N_G(r) = \{s,t,y\}$ (it is allowed to $s = t$). Similarly, as above, let H be a topological subgraph of G and $H \simeq G \backslash \{u,x,y\}$. By the same reason as in Case 1 above, we get that H is a 3-regular connected graph and that $\gamma_M(H) = \gamma_M(G \backslash \{u,x,y\}) = \gamma_M(G) - 1$. Similarly, $|V(H)| < |V(G)|$. By the inductive hypothesis, there exists an independent set $J' \subseteq V(H)$ of H such that $H - J'$ is connected and $|J'| = \gamma_M(H)$. Put $J = J' \cup \{u\}$, we have that J is an independent set of G and $|J| = |J'| + 1 = \gamma_M(H) + 1 = \gamma_M(G)$. Since J' is an independent set of H, at least one of s and t does not belong to J'. Because $H - J'$ is connected, we know that $G - J$ is connected. Hence, the conclusion of the lemma holds for G.

Case 3: *When* (C) *of Lemma* 4 *occurs.* By an analogous argument of the cases above, we easily get that the conclusion of the lemma is valid for G as well.

Hence, by induction, the conclusion of the lemma is obtained. □

We see that Lemma 5 above actually implies that $z(G) \geqslant \gamma_M(G)$, which will be used in the proof of our Theorem 1 in the next section.

Lemma 6. *Let F be a connected graph and $\Delta(F) \leqslant 3$(Δ denotes the maximum degree). A graph G is obtained by this way: add a new vertex $u \notin V(F)$, and join u to some vertices of F such that $d_G(u) = 3$ and $\Delta(G) \leqslant 3$ (for convenience, write $G = F \cup \{u\}$). We have*

(1) *If F is up-embeddable, then G is up-embeddable,*

(2) $\gamma_M(G) \geqslant \gamma_M(F) + 1$.

Proof. Because $d_G(u) = 3$, by the generation of G we have $\xi(G) \leqslant \xi(F)$. If F is up-embeddable, then it implies that G is up-embeddable. Hence, (1) holds. Meanwhile, we notice that $\beta(G) = \beta(F) + 2$. Thus, we have that $\gamma_M(G) = (\beta(G) - \xi(G))/2 \geqslant (\beta(F) + 2 - \xi(F))/2 = \gamma_M(F) + 1$, i.e., (2) holds. □

3. On $\gamma_M(G)$ and $z(G)$

By the above lemmas, we first obtain the following theorem.

Theorem 1. *Let G be a 3-regular connected graph (multiple edges and loops are permitted). Then, we have $\gamma_M(G) = z(G)$.*

Proof. By Lemma 5, we have $z(G) \geqslant \gamma_M(G)$. Now, it suffices to prove $z(G) \leqslant \gamma_M(G)$. Let $z(G) = k$. By the definition of $z(G)$, there exists an independent set $J \subseteq V(G)$ of G such that $G-J$ is connected and $|J| = k$. Let $J = \{u_1, u_2, \ldots, u_k\}$ and let $F = G-J$. Since G is 3-regular and J is an independent set of G, and $F = G - J$ is connected, we may, by successively employing Lemma 6, have

$$\gamma_M(F \cup \{u_1\}) \geqslant \gamma_M(F) + 1$$

$$\begin{aligned}\gamma_M(F \cup \{u_1\} \cup \{u_2\}) &\geqslant \gamma_M(F \cup \{u_1\}) + 1 \\ &\geqslant \gamma_M(F) + 2\end{aligned}$$

$$\vdots$$

$$\begin{aligned}\gamma_M(F \cup \{u_1\} \cup \cdots \cup \{u_{i-1}\} \cup \{u_i\}) &\geqslant \gamma_M(F \cup \{u_1\} \cup \cdots \cup \{u_{i-1}\}) + 1 \\ &\geqslant \gamma_M(F) + i\end{aligned}$$

$$\vdots$$

$$\begin{aligned}\gamma_M(F \cup \{u_1\} \cup \cdots \cup \{u_{k-1}\} \cup \{u_k\}) &\geqslant \gamma_M(F \cup \{u_1\} \cup \cdots \cup \{u_{k-1}\}) + 1 \\ &\geqslant \gamma_M(F) + k.\end{aligned}$$

Hence, we get that $\gamma_M(G) \geqslant \gamma_M(G - J) + k$. It follows that $\gamma_M(G) \geqslant k = z(G)$. Because $z(G) \leqslant \gamma_M(G)$, we have $\gamma_M(G) = z(G)$. Therefore, the proof of the theorem is complete. □

In the proof of Theorem 1, it actually implies the facts that $\gamma_M(G - J) = 0$ and $\gamma_M((G-J) \cup \{u_{j_1}\} \cup \{u_{j_2}\} \cup \cdots \cup \{u_{j_i}\}) = i$ for any $u_{j_1}, u_{j_2}, \ldots, u_{j_i} \in \{u_1, u_2, \ldots, u_k\}$. Hence, we have the following theorem as well.

Theorem 2. *Let G be a 3-regular connected graph and $\gamma_M(G) = k$. Then, for any i, $0 \leqslant i \leqslant k$, there exists a vertex-induced subgraph F of G such that $\gamma_M(F) = i$. Moreover, the number of such F is at least*

$$\binom{k}{i}.$$

A graph G is called a *unicyclic graph* if it is connected, and contains only one circuit, i.e., $\beta(G) = 1$.

In [3], the authors investigate the extensive operations on a 3-regular up-embeddable graph. As a by-product, a structural characterization of a 3-regular up-embeddable graph without cut-edge is presented. Now, we give an analogous result. But here, the restriction 'without cut-edge' may be removed. Thus, the following theorem is viewed as the generalization of a result in [3].

Theorem 3. *Let G be a 3-regular connected graph. Then, G is up-embeddable if and only if the following two conditions are satisfied:*

(1) If $\beta(G) = 0 \pmod 2$, let $|V(G)| = 4k+2$, $k \geqslant 0$, then there exists an independent set $J \subseteq V(G)$ of G and $|J| = k+1$, such that $G - J$ is a tree;

(2) If $\beta(G) = 1 \pmod 2$, let $|V(G)| = 4k+4$, $k \geqslant 0$, then there exists an independent set $J \subset V(G)$ of G, and $|J| = K+1$, such that $G - J$ is a unicylic graph.

Proof. ($\Leftarrow$) Because both a tree and a unicyclic graph are up-embeddable. By Lemma 6, we get that G is up-embeddable. Hence, the sufficiency is obtained.

($\Rightarrow$) If $\beta(G) = 0 \pmod 2$. Because G is 3-regular and $\beta(G) = 0 \pmod 2$, let $|V(G)| = 4k+2$, $k \geqslant 0$. Since G is up-embeddable, we see that $\xi(G) = 0$. Hence, by Theorem 1, we have $z(G) = \gamma_M(G) = [\beta(G) - \xi(G)]/2 = k+1$. Then, there exists an independent set $J \subseteq V(G)$ of G such that $G - J$ is connected. Because $|E(G-J)| = |E(G)| - 3|J| = 3k$ and $|V(G-J)| = |V(G)| - |J| = 3k+1$, we get that $G - J$ is a tree.

If $\beta(G) = 1 \pmod 2$, similarly, let $|V(G)| = 4k+4$, $k \geqslant 0$. Because G is up-embeddable and $\beta(G) = 1 \pmod 2$, we know that $\xi(G) = 1$. By Theorem 1, it follows that $z(G) = \gamma_M(G) = (\beta(G) - 1)/2 = k+1$. Thus, there exists an independent set $J \subseteq V(G)$ of G such that $|J| = k+1$ and $G - J$ is connected. Similarly, because $|E(G-J)| = |E(G)| - 3|J| = 3k+3$ and $|V(G-J)| = |V(G)| - |J| = 3k+3$, we have that $G - J$ is a unicyclic graph.

Thus, this completes the proof of the necessity. □

4. Some lower bounds of $\gamma_M(G)$

The *girth* $g(G)$ of a graph denotes the length of a shortest circuit in G. We call H a *vertex-induced subforest* of a connected graph G if H is a vertex-induced subgraph and H is a forest. Denote $t(G) = \max_H\{|V(H)| \mid H$ is a vertex-induced subforest of $G\}$. A set $F \subseteq V(G)$ is called a *feedback vertex set* (*fvs*) of a connected graph G if $G - F$ is a forest. We denote $f(G) = \min_F\{|F| \mid F$ is an fvs of $G\}$ ($f(G)$ is called the *fvs number* of G in other papers, for [8]). By the definitions of $t(G)$ and $f(G)$, we see that $t(G) + f(G) = |V(G)|$.

Now, we mention some known results on $z(G)$, $f(G)$ and $t(G)$ for a 3-regular connected graph G.

Result 1 (Speckenmeyer [8]). *Let G be a 3-regular connected graph. Then, $f(G) + z(G) = n/2 + 1 (n = |V(G)|)$.*

Result 2 (Speckenmeyer [8]). *Let G be a 3-regular connected graph. Then, $t(G) \geqslant [(3g-3)/(4g-2)]n - (g-1)/(2g-1)$ $(n = |V(G)|, g = g(G))$.*

In the case of $g(G) = 3$, a best possible lower bound of $t(G)$ is the following.

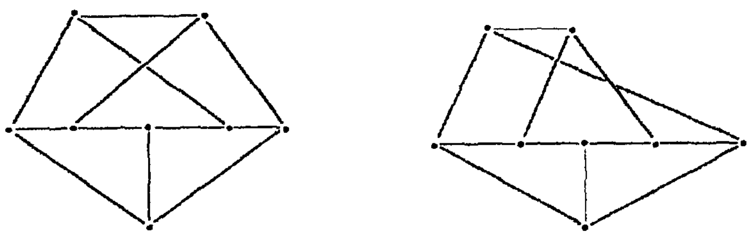


Fig. 3.

Result 3 (Speckenmeyer [7]). *Let G be a 3-regular connected graph and $g(G) = 3$. Then, $t(G) \geqslant \frac{5}{8}n - 1$ ($n = |V(G)|$).*

If $g(G) > 3$, a best possible lower of $t(G)$ follows

Result 4 (Zheng and Liu [12]). *Let G be a 3-regular connected graph and $g(G) > 3$. If $G \neq G_1$ and G_2 (shown in Fig. 3), then $t(G) \geqslant \frac{2}{3}n$ ($n = |V(G)|$).*

According to the above results and our Theorem 1, we can establish some new lower bounds of $\gamma_M(G)$ for a 3-regular connected graph G. Hence, we have the following theorem.

Theorem 4. *Let G be a 3-regular connected graph and let g be the girth of G and $n = |V(G)|$. We have*

(1) $\gamma_M(G) \geqslant (g-2)n/(4g-2) + g/(2g-1)$;

(2) *If $g = 3$, then $\gamma_M(G) \geqslant n/8$;*

(3) *If $g > 3$, $G \neq G_1$ and G_2 (G_1 and G_2 shown in Fig. 3), then $\gamma_M(G) \geqslant n/6 + 1$.*

Proof. By the definitions of $t(G)$ and $f(G)$, we notice that $t(G) + f(G) = n$. Combining Theorem 1, $\gamma_M(G) = z(G)$, together with Results 1–4 above, the theorem is immediately obtained. □

Now we give a result on the up-embeddablity in terms of the girth.

Corollary. *Let G be a 3-regular connected graph and g be the girth of G. Then $\lim_{g\to\infty} \gamma_M(G)/\lfloor \beta(G)/2 \rfloor = 1$. In other words, when g is large enough, G is up-embeddable.*

Proof. By Theorem 4(1) above, we have

$$\gamma_M(G) \Big/ \left\lfloor \frac{\beta(G)}{2} \right\rfloor \geqslant \left(\frac{(g-2)n}{4g-2} + \frac{g}{2g-1} \right) \Big/ \left\lfloor \frac{n+2}{4} \right\rfloor \quad (n = |V(G)|).$$

Then, $\lim_{g\to\infty} \gamma_M(G)/\lfloor\beta(G)/2\rfloor \geqslant 1$ (when $g \to \infty$, $n \to \infty$). By the definition of $\gamma_M(G)$, we have $\gamma_M(G)/\lfloor\beta(G)/2\rfloor \leqslant 1$. Thus, $\lim_{g\to\infty} \gamma_M(G)/\lfloor\beta(G)/2\rfloor = 1$. Hence, the corollary follows. □

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